Physics of the Quantum Universe

Test 1 (online)

20 November 2020

Exam instructions

Dear students

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Good luck. Diederik

1 Diffraction Pattern (3 points)

Consider a material with a simple cubic lattice with lattice spacing equal to 3 Angstrom. You are illuminating this material with photons that give rise to a diffraction pattern. Consider an angle equal to (in degrees) the last two digits of your student number (clearly indicate these numbers in your answer) - where an angle of zero degree corresponds to bouncing straight back. What is the longest wavelength (of the incident photons) for which you see negative interference at this angle?

2 Electron Capture (3 points)

Consider a hypothetical ionized atom X^+ (which is one electron short of being neutral) in its ground state. This ion captures an incoming electron with zero (or negligible) kinetic energy, and in the process emits two photons whose wavelength (in nm) are given by the first three digits of your student number (photon 1) and the last three digits of your student number (photon 2). Clearly indicate these numbers in your answer. What is the binding energy in eV) of this outermost electron?

3 Bohr Model (3 points)

Two crucial developments in the early stages of quantum theory are Planck's derivation of the blackbody spectrum and Bohr's derivation of the hydrogen spectrum. As discussed during the lectures, the classical limit of the blackbody spectrum yields the Raleigh-Jeans formula which is correct at large wavelength but wrong for short wavelength. In this question you are asked to think about the classical limit of the Bohr model. In particular, address the following two points: what happens to the allowed radii at which the electron can orbit and what are the possible frequencies that it can emit? Answer these questions both using (very brief) physical reasoning and by outlining what happens to the expressions for the admissible radii and energies in the Bohr model when sending Planck's constant to zero.

$d = 3Å = 3 \times 10^{-10} m$ $\rightarrow \lambda = 3 \times 10^{-10} m$

$\theta = 70°$ (student no [redacted])

~~$1 - \cos\theta = 0.65798$~~

$d \sin\theta = \frac{2n+1}{2}\lambda$ → for destructive interference / if $n=0$, → λ longest when n smallest.

$d\sin\theta = \frac{\lambda}{2} \rightarrow \lambda = 2d\sin\theta = 5.64 \times 10^{-10}$ m. = wavelength of shifted photons

wavelength of incident photons = λ'

→ $\theta = 180 - 70 = 110°$

$\Delta\lambda = \lambda_c(1 - \cos\theta) =$ ~~1.5989×10^{-12} m~~ 3.26×10^{-12}.

$\lambda' = \lambda - \Delta\lambda =$ ~~5.622×10^{-10}~~ **5.61×10^{-10} m**

X^+

photon 1: 425 nm
photon 2: 170 nm → where $h = 6.626 \times 10^{-34}$ Js and $c = 3 \times 10^8$ m/s

$E_1 = hf_1 = \frac{hc}{\lambda_1} = 4.677 \times 10^{-19}$ J $= 2.923$ eV

$E_2 = hf_2 = \frac{hc}{\lambda_2} = 1.1693 \times 10^{-18}$ J $= 7.308$ eV

Binding energy $= E_1 + E_2 = 10.231$ eV

For the hydrogen atom,

$r_n = a_0 n^2$ where $a_0 = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}$

This arises from the quantization of angular momentum,

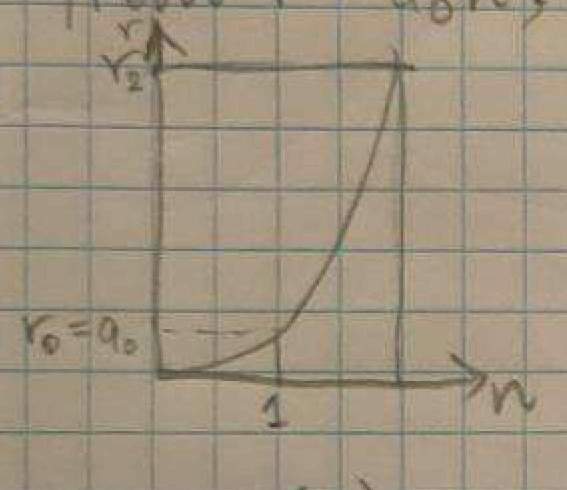
i.e. $m_e v_n r_n = n\hbar$

Physical reasoning for classical limit:

if energy levels become continuous, electrons (which are seen as accelerating particles around the nucleus) would lose energy and spiral into the nucleus, which we know does not happen.

The permitted ranges of frequencies it can emit while transitioning to a different energy would also become continuous, but we know only photons with discrete energies can be absorbed (eg. by photoelectric effect experiment)

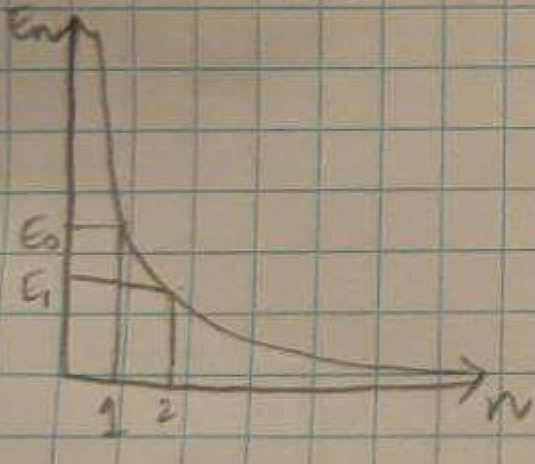
from $r = a_0 n^2$, the graph of n vs r can be seen as,



→ where only discrete values of r can be present, corresponding to $n = 1, 2$ ---

But with classical theory, any value of r between these discrete states become permitted.

$E_n = -E_0\left(\frac{1}{n^2}\right)$



Quantum: only discrete values of E_n allowed

Classical: any value of E_n between these discrete values permitted.

Physics of the Quantum Universe

Test 2 (online)

04 December 2020

Exam instructions

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Good luck. Diederik

1 Quantum Tunneling (3 points)

X- the first two digits of your student number (if zero, take 34)
Y- the last two digits of your student number (if zero, take 45)

Define Max as the highest of these two numbers and Min as the smallest (if equal, increase Max by 10) Imagine an electron coming in from the left with an amount of Min eV kinetic energy that runs into a barrier that is Max eV high and 1 nm wide. We are interested in the probability that the particle tunnels through this barrier.

a) What is the probability that the electron tunnels through this barrier?

b) Qualitatively describe what happens if the particle would be a proton instead, would the probability of tunnelling through be higher or lower?

2 Particle in a Box (3 points)

Consider a quantum particle in a box in its ground state.

a) Find the expectation value of momentum for the ground state of the quantum particle in a box in terms of eg, Planck's constant, the mass m and the width of the box

b) if we were to measure the momentum of the particle in this state, would this always coincide with your answer at a)? Briefly explain why (not).

3 Heisenberg Uncertainty (3 points)

X- the first two digits of your student number (if zero, take 34)
Y- the last two digits of your student number (if zero, take 45)

A certain atom is measured to have an excited state that is an amount of X ev above the energy of the ground state. Moreover, it generically remains in this excited state for a period of V ns before transitions to the ground state.

a) Briefly explain what the Heisenberg uncertainty principle implies for the accuracy with which we can measure the energy of an unstable state.

b) Estimate the uncertainty in the frequency of the photon that is emitted during this quantum transition.

Text submission:

Q1- see image.

Q2- (a) see image. Note: after establishing the integral, one cannot use the odd/even integrand trick unless the integrand bounds extend evenly into the positive and negative. The value of the integral still turns out to be zero, as an electron has equal probability to be moving left or right at a certain instant.

(b) You do not *necessarily* measure 0, but you can also measure other values. The expectation value is the *average* value for many identical measurements, *not* the expected value. Also, there is always some uncertainty in the momentum distribution, so measuring a particular value of momentum is never guaranteed.

Q3- (a) the energy-time Heisenberg uncertainty relation implies that measuring the value of either energy or time precisely sacrifices the accuracy with which we know the value of the other. A minimum uncertainty of h(bar)/2 will always exist in the product of the uncertainties in energy and time, and this is not so due to inadequacies in the measurement devices but is rather a natural uncertainty that occurs in quantum mechanics. For an atom in an unstable state, this implies that the smaller the uncertainty in time (1.e. the quicker it decays), the greater the uncertainty in energy

(extra: for situations such as particle-antiparticle pair production, the pair of particles produced essentially borrow the energy from 'nowhere', as is allowed by Heisenberg if the time period for which they exist before mutually annihilating is small enough. in the uncertainty principle here, the uncertainty in energy can be seen as its total value, as it was initially zero. this would imply that the shorter the length of time they exist for, the greater energy they can have for that time period)

(b) see image

Q1 (a) X = 42
Y = 70

Max = 70, min = 42

$E = 42\ eV$ $\quad U_0 = 70\ eV$ $\quad L = 10^{-9}\ m$ $\quad m = 9.11 \times 10^{-31}\ kg$

$\downarrow \qquad\qquad \downarrow$

$= 6.72 \times 10^{-18}\ J \qquad = 1.12 \times 10^{-17}\ J$

$\beta = \frac{\sqrt{2m(U_0 - E)}}{\hbar} = 2.71 \times 10^{10}\ m^{-1}$ ($\gg 1$), so we can use the approximation.

$T(L, E) = 16\left(\frac{42}{70}\right)\left(\frac{28}{70}\right)e^{-2\beta L} = 1.154 \times 10^{-23}$

$\approx 1.2 \times 10^{-23}$, or $(1.2 \times 10^{-21})\%$

(b) $m_p \approx 1836 m_e$

Since mass of proton is significantly higher than mass of electron,

$\beta_{(p)}$ would be greater than $\beta_{(e)}$

Hence in the formula for tunnelling probability,

$e^{2\beta L}$ in the denominator would increase, causing the overall value of $T(L, E)$ to decrease (exponentially).

Q2 Considering the case of a free particle,

$\psi(x) = A\sin(kx) + B\cos(kx)$

Let L = width of the box, then imposing boundary conditions,

$\psi(x) = 0$ at $x = 0$ and at $x = L$

→ since $\cos(0) = 1$, ψ must be $A\sin(kx)$.

at $x = L$, $A\sin(kx)$ must be 0 so $kx = n\pi \Rightarrow kL = n\pi$

$\Rightarrow k = \frac{n\pi}{L}$

so, $\psi = A\sin\left(\frac{n\pi x}{L}\right)$

to find the normalization constant A, use $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

since $\psi = 0$ when $x \leq 0$ and $x \geq L$, this can be rewritten as

$$\int_0^L |\psi|^2 dx = 1 \rightarrow \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1 \rightarrow A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

using double angle identity,

$$\frac{A^2}{2}\int_0^L \left(\cos\left(\frac{2n\pi x}{L}\right) + 1\right) dx = \frac{A^2}{2}\left[\frac{\sin\left(\frac{2\pi n x}{L}\right)}{\frac{2n\pi}{L}} + x\right]_0^L = \frac{A^2}{2}(0 + L - 0) = 1$$

$$\rightarrow A^2 = \frac{2}{L} \rightarrow A = \sqrt{\frac{2}{L}} \rightarrow \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$\langle p \rangle = \int_0^L \psi^* \left(-i\hbar \frac{\partial}{\partial x} \psi\right) dx \qquad \frac{\partial}{\partial x}\psi = \frac{\partial}{\partial x}\left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right) = \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right)\left(\frac{L}{n\pi}\right)$$

$$= -i\hbar \int_0^L \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right)\left(\sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right)\left(\frac{L}{n\pi}\right)\right)$$

$$= -i\hbar \left(\frac{2}{L}\right)\left(\frac{L}{n\pi}\right) \int_0^L \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx$$

Since inside the integral is the product of an odd and even function, the value of the integral is an odd function and becomes 0 by symmetry.

so, $\langle p \rangle = 0$

this makes sense as approximately an equal number of particles would be moving left and right at a given time.

$X = 42 \quad Y = 70$

$E_n = 42eV + E_1 \qquad \Delta t = 70ns = 70 \times 10^{-9}\,s$

(b) Energy released during transition $= E_n - E_1 = 42eV$

$42eV = 6.72 \times 10^{-18}\,J = hf$

$f = 1.0142 \times 10^{16}\,Hz$

now, assume that for ground state, $\Delta E \Delta t = \frac{\hbar}{2}$

then $\Delta E = \frac{\hbar}{2\Delta t} = 7.5357 \times 10^{-28}\,J = h\Delta f$

$\Delta f = \frac{\Delta E}{h} = 1.1373 \times 10^{6}\,Hz$

Hence, $\boxed{\Delta f \approx 1.1 \times 10^{6}\,Hz}$

and $\frac{\Delta f}{f} = \frac{1.1373 \times 10^{6}\,Hz}{1.0142 \times 10^{16}\,Hz} = 1.1214 \times 10^{-10} \approx 1.1 \times 10^{-10}$

Physics of the Quantum Universe

Test 3 (online)

18 December 2020

Exam instructions

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Good luck. Diederik

1 Spin (3 points)

Consider a hypothetical atomic element that has no orbital angular moment and has a spin that equals one unit (s=1).
(a) In a Stern-Gerlach experiment with a magnetic field that is orthogonal to the motion of the atoms, into how many bands would a beam of these elements split up?
(b) What are the values of the magnetic dipole moment of this particle for the possible outcomes of the previous question?
(c) Given a known magnetic field, can one observationally tell the difference between elements with one unit of angular momentum (and no spin) and elements with one unit of spin (and no angular momentum) using the Stern-Gerlach set-up? Briefly explain your answer.

2 Atomic Structure (3 points)

(a) Briefly explain the orbital structure of the electrons of Lithium; which quantum states do the three electrons occupy?
(b) Briefly explain what is special about the orbital structure of the Z=19 element K (Potassium) of the periodic table.
(c) What would be the orbital structure in terms of 1s etc) of the Z=10 element if an electron would have spin -1 instead of spin -1/2?

3 Wavefunctions (3 points)

Consider the n=2 wavefunctions of the electron in a Hydrogen atom. In this question you can neglect the spin of the electron.
(a) How many distinct quantum states are possible for n=2? Indicate the possible values of the relevant quantum numbers.
(b) Which n=2 wavefunctions are spherically symmetric? Briefly explain your answer.
(c) At which distance from the nucleus are you most likely to find an electron if it is in the n=2, l=1 quantum state?

Q1. Spin

(a) For a particle with spin 1, there are three possible integer states for $-l < m_s < l$, i.e. -1, 0 and 1. So, the Stern-Gerlach experiment would produce 3 bands.

(b) See image.

$S = 1$

(b) $\vec{\mu}_s = \frac{-e}{m_e}\vec{S}$

$\vec{S}_z = m_s\hbar = \hbar \quad (m_s = 1)$

so $\vec{\mu}_{sz} = \frac{-e}{m_e}\vec{S}_z = \frac{-e\hbar}{m_e} = -2\mu_B$

$= -2(9.27 \times 10^{-24}\ J/T) = -1.94 \times 10^{-23}\ J/T$

Q2. Atomic Structure

(a) A neutral atom of Li has 3 electrons, 2 of which occupy the n=1 energy level and the 3rd occupies n=2. SEE IMAGE for quantum numbers. For the n-2 electron, the l=0 state is preferred over the l=1 because its probablilty function has a higher chance of seeing an unscreened nucleus (r<bohr radius) and hence having a higher binding energy.

Li $1s^2\ 2s^1$ (electronic configuration)

e_1^- : $n=1$, $l=0$, $m_l=0$, $m_s=+1/2$ (+ by convention)
e_2^- : $n=1$, $l=0$, $m_l=0$, $m_s=-1/2$ (opp to $m_s(e_1^-)$)
e_3^- : $n=2$, $l=0$, $m_l=0$, $m_s=+1/2$ (again, by convention we choose +. In reality it could be + or −)

(b) The electron configuration of K is given by 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 45^1. This seems special because the 4s orbital starts to be filled before the 3d orbital (and n=3<n=4, so one would expect the entire n=3 shell to be filled before n=4 starts to (this would be the case if there was no screening effect). However, this happens because after the 3p orbital is filled, electron-electron repulsion gets so strong that n=4 l=0 (4s) is energetically favoured over n=3 l=2 (3p).

(c) Pauli's exclusion principle states that no two electrons in an atom can have the same values for all 4 quantum numbers. This is the reason that for electrons with spin +/-0.5, a single orbital can have at most two electrons with opposing spins. This would change if the value of electron spin would become +/-1, as an orbital could then contain 3 electrons, with spins -1, 0 and 1. In that case, the orbital structure would be 1s^3, 2s^3, 2p^4.

Q3. Wavefunctions

(a) See image.

→ NEGLECTING SPIN

(a) 4 distinct quantum states are possible for n=2.
The possible values for l are 0 and 1.
- for $l = 0$, $m_l = 0$
- for $l = 1$, $m_l = -1, 0, +1$

Hence for the quantum states a, b, c and d,

	n	l	m_l
a	2	0	0
b	2	1	-1
c	2	1	0
d	2	1	1

(if m_s were also to be considered, each state would correspond to 2 states, one with $m_s = +1/2$ and one with $m_s = -1/2$ → hence total number of distinct quantum states would be 8.)

(b) The wavefunction for n=2 l=0 is spherically symmetric. The wavefunctions corresponding to n=2 l=1 are not. Mathematically, this is because the wavefunctions of l=0 depends on only the radial distance from the nucleus (which is why s (l=0) orbitals are spherical) while the wavefunctions for l=1 also depend on the spherical coordinate theta.

(c) The most probable value is the value of r when the derivative of the probability function is set to zero. This comes out to be $4a_0$

Physics of the Quantum Universe

Test 4 (online)

15 January 2021

Exam instructions

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1 Strong Force (3 points)

1. Briefly explain (in at most two sentences) why there needs to be a powerful nuclear force in order for stable nuclei to exist.
2. Using the Heisenberg uncertainty principle, determine the range of the strong force if this force is produced by the exchange of pions.
3. Draw the Feynman diagram corresponding to the exchange of a neutral pion between two protons. Clearly indicate the quark content of the nuclei and pion.

2 Standard Model (3 points)

1. For each of the three forces in the Standard Model, indicate what the corresponding force particles are.
2. Can the following kaon decay process take place: $(K^{+})- > (\mu^{+}) + (\nu_{\mu})$
3. The matter of the Standard Model is divided into quarks and leptons. Does the Standard Model contain a force that can convert a quark into a lepton? Briefly explain your answer in at most two sentences.

3 Big Bang (3 points)

1. On average, how far away are galaxies that are moving away from us at 2 percent of the speed of light?
2. Briefly explain (in at most two sentences) what observed property of the Cosmic Microwave Background was seen as strong evidence for the Big Bang.
3. Briefly explain (in at most two sentences) which observation led to the introduction of Dark Energy in our understanding of the Universe.

Q1. Strong Force

(a) Nuclei consist of only protons and neutrons, and since like charges are electrostatically repelled, protons within a nucleus should not be able to exist so close together (on the order of magnitude of a femtometer) due to electrostatic repulsion. The strong force (attractive), however, is stronger than the (repulsive) electrostatic force at this range and hence it keeps the nucleons bound together and hence ensures the stability of the nucleus.

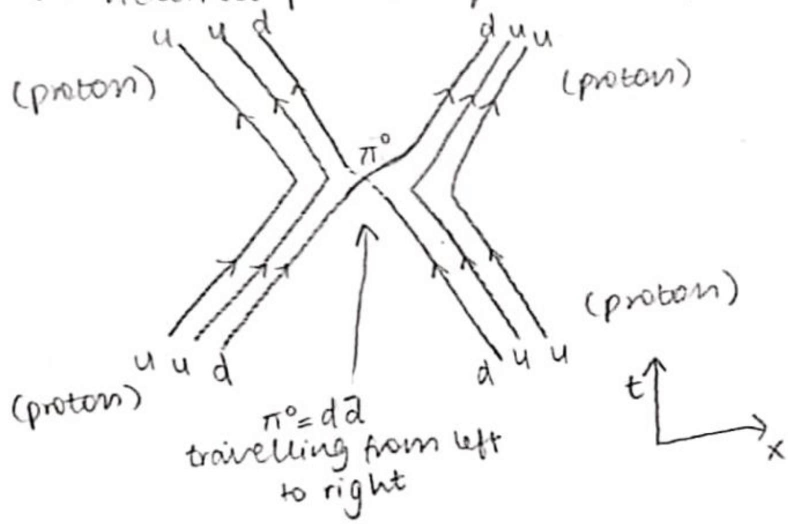
(b) and (c): see image

(b) $\underbrace{\Delta E}_{mc^2}\,\underbrace{\Delta t}_{\frac{d}{c}} \approx \frac{\hbar}{2}$ → mc^2 of pion = 139.6 MeV (assuming $\pi^{\pm}$) (would differ slightly for π^0 (135.0 MeV) but same order of magnitude in answer)

$$\hookrightarrow d = \frac{\hbar c}{2mc^2} = \frac{(6.582\times10^{-16}\ \text{eV}\cdot\text{s})(3\times10^{8}\ \text{m/s})}{2\times139.6\times10^{6}\ \text{eV}}$$

$$= 7.07\times10^{-16} \approx 10^{-15}\ \text{m}$$

(c) neutral pion has quark composition $u\bar{u}$ or $d\bar{d}$



Q2. Standard Model

(a) and (b): see image

(c): Interactions within the standard model require certain conservation laws to be followed, including the conservation of baryon number and lepton number, and it would violate these laws and hence not be possible within the standard model for a quark to simply decay into a lepton on its own (assuming isolated quarks would exist in the first place). However, it is possible for an interaction between a quark and an antiquark to produce a lepton and it's antineutrino (or vice versa) (since initial and final baryon and lepton numbers would all be +1-1=0) via the weak force for example, thus effectively converting quark to lepton.

(a) Force | Mediating particles: | Acts on:

Force	Mediating particles	Acts on
Electromagnetic:	Photon	charged particles (quarks, leptons but not neutrinos)
strong	gluon	colour (quarks only)
weak	W^+, W^-, Z^0 bosons.	quarks, leptons

(b) $K^+ \longrightarrow \mu^+ + \nu_\mu$

lepton no.	$0 \longrightarrow -1 + 1 = 0$
baryon no.	$0 \longrightarrow 0 + 0 = 0$
charge	$1 \longrightarrow 1 + 0 = 1$
strangeness	$1 \longrightarrow 0 + 0 = 0$

strangeness is not conserved, so the decay can occur only via the weak force.

Q3. Big Bang

(a): see image

(b): CMB is seen to exhibit a *blackbody spectrum* (with a current peak wavelength that corresponds to a temperature of 2.7K, accounting for the universe's expansion) , which is only emitted by things in thermal equilibrium and at high temperatures, indicating that the universe *has experienced a phase of very high temperature, when CMB was emitted*. Going back in time to when the (expanding) universe was smaller (and hence hotter), CMB can be seen as *remnant heat from the Big Bang*.

(c) Observations have indicated that the expansion rate of the universe is increasing (i.e. it is an accelerating universe), however the only known force that is effective at cosmological distances is the gravitational force, which is an attractive force and should hence be pulling matter closer together, slowing down the expansion rate and hence leading to a decelerating universe, contrary to observational proof. The only explanation to this is that there is some other unknown component responsible for the acceleration of the universe's expansion, and this is said to be called 'Dark Energy'.

(a) $v = H_0 d \rightarrow d = \frac{v}{H_0} = \frac{0.02 \times 3 \times 10^5 \text{ km/s}}{70 \text{ km/s / Mpc}} = 85.7 \text{ Mpc} \approx 86 \text{ Mpc}$